

All India Science Teachers' Association, West Bengal

Preparatory Test for Entrance Examinations 2021

Class-XII, Paper-III (Mathematics)

Four probable answers are provided to each question. Select the best answer as correct answer. Each correct answer will be awarded +4 marks. Each wrong answer will be awarded -1 mark. In case of no response, 0 mark will be awarded.

- $x = \cos\alpha + i\sin\alpha$, $y = \cos\beta + i\sin\beta$, $z = \cos\gamma + i\sin\gamma$ and $x+y+z=0$ then value of $(\frac{1}{x} + \frac{1}{y} + \frac{1}{z})$ is
(If $x = \cos\alpha + i\sin\alpha$, $y = \cos\beta + i\sin\beta$, $z = \cos\gamma + i\sin\gamma$ and $x+y+z=0$ then value of $(\frac{1}{x} + \frac{1}{y} + \frac{1}{z})$ is)
a. xyz
b. 1
c. $\operatorname{Re}(x) + \operatorname{Re}(y) + \operatorname{Re}(z)$
d. 0
- $e^{\sin x} - e^{-\sin x} - 4 = 0$, number of real values of x is
a. 0
b. 1
c. 2
d. 3
- The four sides of a quadrilateral are given by $xy(x-5)(y-4)=0$. The equation of the line that divides the quadrilateral in two equal areas and parallel to $x-3y=0$ is
a. $2x-6y-7=0$
b. $2x-6y+7=0$
c. $2x-6y+17=0$
d. $2x-6y-17=0$
- $(\frac{1}{x} + x^{\log_{10} x})^5$ has a term independent of x is 1000. If the third term in the expansion $(\frac{1}{x} + x^{\log_{10} x})^5$ be 1000 then values of x is
a. 100, $\frac{1}{\sqrt{10}}$
b. 300, $\frac{1}{\sqrt{10}}$
c. 10, $\frac{1}{\sqrt{10}}$

- d. $500, \frac{1}{\sqrt{10}}$
5. $2a+3b+6c=0$ ☐ ☐ $ax^2+bx+c=0$ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ (If $2a+3b+6c=0$, then one root of the equation $ax^2+bx+c=0$ lies in the interval)
- a. $2 < x < 3$
b. $1 < x < 2$
c. $0 < x < 1$
d. $1 < x < 3$
6. $\int_0^\pi |\sin x + \cos x| dx$ ☐ ☐ ☐ (Value of $\int_0^\pi |\sin x + \cos x| dx$ is)
- a. 2
b. $\sqrt{2}$
c. $2\sqrt{2}$
d. 4
7. $5^{\log_e x} = 50 - x^{\log_e 5}$ ☐ ☐ ☐ ☐ (If $5^{\log_e x} = 50 - x^{\log_e 5}$, then the value of x is)
- a. 10
b. e
c. 100
d. e^2
8. $f(x) = \cos(\sin x) + \sin(\cos x)$ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ (The fundamental period of the function $f(x) = \cos(\sin x) + \sin(\cos x)$ is)
- a. π
b. $\frac{\pi}{2}$
c. 2π
d. $\frac{2\pi}{3}$
9. $A = \{(x, y) \in \mathbb{R}^2: y = e^x\}$ ☐ ☐ $B = \{(x, y) \in \mathbb{R}^2: y = x\}$ ☐ ☐ $A \cap B$ ☐ ☐ (If $A = \{(x, y) \in \mathbb{R}^2: y = e^x\}$ and $B = \{(x, y) \in \mathbb{R}^2: y = x\}$ $A \cap B$ will be)
- a. ϕ
b. ☐ ☐ ☐ ☐ ☐ ☐ (a singleton set)
c. ☐ ☐ ☐ ☐ ☐ ☐ (an infinite set)
d. $\{\phi\}$
10. a, b, c ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ $a+b+c=bx$ ☐ ☐ (If a, b, c be in G. P. and $a+b+c=bx$, then)
- a. $-1 < x < 3$

b. $-1 \leq x \leq 3$

c. $x > 3$ or $x < -1$ ($x > 3$ or $x < -1$)

d. $x \geq 3$ or $x \leq -1$ ($x \geq 3$ or $x \leq -1$)

11. 1, 3, 9 ... 100 and 1, 3, 7, ... 100 (The number of terms which are equal in the two progressions 1, 3, 9 ... to 100 terms and 1, 3, 7, ... to 100 terms is)

a. 3

b. 5

c. 2

d. 4

12. $0 \leq x \leq \frac{\pi}{2}$ (The number of roots of the equation $x^2 + \sin^2 x = 1$ in $0 \leq x \leq \frac{\pi}{2}$ is)

a. 0

b. 3

c. 2

d. 1

13. Numbers lying between 100 and 1000 and divisible by 3 and 5 are to be formed with the digits 1, 2, 3, 4, 5 without repetitions. Total number of such numbers is)

a. 24

b. 36

c. 12

d. 32

14. $\sin 7x + \cos 2x = -2$ ($0 \leq x \leq \frac{\pi}{2}$) (The least difference between the roots in the first quadrant ($0 \leq x \leq \frac{\pi}{2}$) of the equation $\sin 7x + \cos 2x = -2$ is)

a. 0

b. $\frac{2\pi}{7}$

c. $\frac{3\pi}{7}$

d. $\frac{\pi}{14}$

15. If $f(x) = x\sqrt{e^{x(4-3x)}}$ (If $f(x) = x\sqrt{e^{x(4-3x)}}$, then)

19. $\int \frac{dx}{x^2+a}$ ($a < 0$) ☐ ☐ ☐ ☐ (Value of $\int \frac{dx}{x^2+a}$ ($a < 0$) is)
- a. $\frac{1}{\sqrt{a}} \tan^{-1} \frac{x}{\sqrt{a}} + c$
- b. $\frac{1}{\sqrt{-a}} \tan^{-1} \frac{x}{\sqrt{-a}} + c$
- c. $\frac{1}{2\sqrt{-a}} \log \left| \frac{x-\sqrt{-a}}{x+\sqrt{-a}} \right| + c$
- d. $\frac{1}{2\sqrt{a}} \log \left| \frac{x-\sqrt{a}}{x+\sqrt{a}} \right| + c$

20. $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ cut the coordinate axes in four concyclic points, then

- a. $a_1b_1 = a_2b_2$
- b. $a_1a_2 = -b_1b_2$
- c. $a_1b_2 = -b_1a_2$
- d. $a_1a_2 = b_1b_2$

21. $\lim_{x \rightarrow \infty} \left(\frac{4x^2+1}{4x^2-1} \right)^{\frac{x^3}{1+x}}$ (Value of $\lim_{x \rightarrow \infty} \left(\frac{4x^2+1}{4x^2-1} \right)^{\frac{x^3}{1+x}}$)

- a. $\sqrt{e}$
- b. 1
- c. e^2
- d. e

22. $f(1)=2$ and $\lim_{x \rightarrow 1} \int_1^{f(x)} \frac{2t}{x-1} dt = 4$ then $f'(1)$ will be)

(Let f be a differentiable function and $f(1)=2$. If $\lim_{x \rightarrow 1} \int_1^{f(x)} \frac{2t}{x-1} dt = 4$ then $f'(1)$ will be)

- a. 1
- b. 2
- c. 4
- d. $\frac{1}{2}$

23. $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ (The locus of the vertices of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is)

(The locus of the vertices of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is)

- a. $xy = \frac{35}{16}$
- b. $xy = \frac{36}{105}$
- c. $xy = \frac{105}{64}$
- d. $xy = \frac{3}{4}$

24. $f(x) = {}^{7-x}P_{x-3}$ (The range of the function $f(x) = {}^{7-x}P_{x-3}$ is)

- a. $\{x: x \in \mathbb{N}, x \leq 6\}$
- b. $\{1, 2, 3\}$
- c. $\{x: x \in \mathbb{N}, x < 6\}$
- d. $\{1, 2, 3, 4\}$

25. $y=f(x)$ is a function symmetric about the line $x=2$ (If the graph of the function $y=f(x)$ be symmetric about the line $x=2$ then)

- a. $f(x) = -f(-x)$
- b. $f(x+2) = f(x-2)$
- c. $f(x) = f(-x)$
- d. $f(2+x) = f(2-x)$

26. In a parabola $x^2=16y+2$, PSP' (S being the focus of the parabola) is a focal cord. If $\overline{SP}=16$, $\overline{SP'}$ will be equal to

(In a parabola $x^2=16y+2$, PSP' (S being the focus of the parabola) is a focal cord. If $\overline{SP}=16$, $\overline{SP'}$ will be equal to)

- a. $\frac{8}{3}$
- b. 8
- c. 16
- d. $\frac{16}{3}$

27. $\frac{x^2}{4} + \frac{y^2}{1} = 1$ meets the curve $x^2 + \frac{y^2}{a^2} = 1$ in four distinct points and if $a=b^2-10b+25$, then

(If the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ meets the curve $x^2 + \frac{y^2}{a^2} = 1$ in four distinct points and if $a=b^2-10b+25$, then)

- a. $b < 4$ or $b > 6$ ($b < 4$ or $b > 6$)
- b. $b \leq 4$, $b \geq 6$
- c. $4 < b < 6$
- d. $4 \leq b \leq 6$

28. $\lim_{n \rightarrow 0} \tan\left(\frac{\pi}{4} + x\right)^{\frac{1}{x}} =$

- a. e
- b. e^2
- c. e^3
- d. e^5

29. $\theta = \alpha$ (The Cartesian equation of $\theta = \alpha$ is)

- a. $y = x \sin \alpha$
- b. $y = \cos \alpha$
- c. $y = x \tan \alpha$
- d. $y = x \cot \alpha$

30. P(1,2) and Q(3,4) are two points. R is a point on the line segment PQ such that

$\overline{PR} + \overline{RQ}$ is minimum. (Let P(1,2) and

a. $(\frac{3}{5}, 0)$
b. $(\frac{5}{3}, 0)$
c. $(-\frac{3}{5}, 0)$
d. $(-\frac{5}{3}, 0)$

- $\frac{e^{x-1}}{x} > 1$
- $\frac{x}{e^{x^2-1}} > 1$
- $\frac{x}{e^{1-x}} < 1$
- $\frac{e^{x-1}}{x} < 1$

- 6
- 5
- 8
- 7

- 0
- $\frac{1}{2}$
- $-\frac{1}{2}$
- 1

- 23
- 9
- 207

d. 19

35. $\alpha = \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}$ ☐ ☐ ☐ ☐ $\left(\frac{1+\alpha}{2}\right)^{3n}$ ☐ ☐ ☐ ☐ (If $\alpha = \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}$, then $\left(\frac{1+\alpha}{2}\right)^{3n}$ is equal to)

a. $(-1)^n$

b. $(-i)^n$

c. $\frac{1}{2^{3n}}$

d. $\frac{(-1)^n}{2^{3n}}$

36. $y^2 + \left(x - \frac{1}{y}\right) \frac{dy}{dx} = 0$ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ (The general solution of the differential equation $y^2 + \left(x - \frac{1}{y}\right) \frac{dy}{dx} = 0$ is)

a. $x = \frac{1}{y} + 1 + Ce^{\frac{1}{y}}$

b. $x = 1 - \frac{1}{y} + Ce^{-\frac{1}{y}}$

c. $x = 1 + \frac{1}{y} + Ce^{\frac{-1}{y}}$

d. $x = 1 - \frac{1}{y} + Ce^{\frac{1}{y}}$

37. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin x}{x} dx$ ☐ ☐ ☐ ☐ (Value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin x}{x} dx$ is)

a. < 0

b. $> \frac{\sqrt{2}}{6}$

c. $\leq \frac{\sqrt{2}}{6}$

d. 1

38. $\frac{\cos x}{\cos y} = \frac{a}{b}$ ☐ ☐ ☐ ☐ $a \tan x + b \tan y$ ☐ ☐ ☐ ☐ (If $\frac{\cos x}{\cos y} = \frac{a}{b}$, then value of $a \tan x + b \tan y$ will be)

a. $(a+b)\left(\tan \frac{x}{2} + \tan \frac{y}{2}\right)$

b. $(a+b)\tan \frac{x+y}{2}$

c. $(a+b)\cot \frac{x+y}{2}$

d. $(a+b)\left(\cot \frac{x}{2} + \cot \frac{y}{2}\right)$

39. $\sum_{k=1}^{k=\infty} \cot^{-1} \left(k^2 + \frac{3}{4}\right)$ ☐ ☐ ☐ ☐ (Value of $\sum_{k=1}^{k=\infty} \cot^{-1} \left(k^2 + \frac{3}{4}\right)$ is)

a. $\frac{\pi}{2}$

- b. $-\frac{\pi}{2}$
 c. $\cot^{-1} \frac{1}{2}$
 d. $\cot^{-1} 2$

40. $\{f(x)\}^n = f(nx) \forall x$ ☐ ☐ ☐ ☐ $f(x).f(nx)$ ☐ ☐ ☐ ☐ (If $\{f(x)\} = f(nx) \forall x$, then $f(x).f(nx)$ is equal to)
 a. x^n
 b. $n.x^{n-1}$
 c. $f(x)$
 d. $f(x).f(nx)$

41. $\int x(x^x)^x (2\log x + 1)dx$ ☐ ☐ ☐ ☐ (Value of $\int x(x^x)^x (2\log x + 1)dx$ is)
 a. $(x^x)^x + C$
 b. $\frac{(x^x)^x}{\log(x^x)} + C$
 c. $x^{x^2+1} \cdot \log x + C$
 d. $(x^x)^x + C$

42. $a > 0$ ☐ ☐ $y^2 = 4a(x+a)$ ☐ $y^2 = -4a(x-a)$ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ (The area (in sq. units) bounded by the curves $y^2 = 4a(x+a)$ and $y^2 = -4a(x-a)$, $a > 0$ is)
 a. $\frac{2}{3}a^2$
 b. $\frac{4}{3}a^2$
 c. $\frac{8}{3}a^2$
 d. $\frac{16}{3}a^2$

43. $A \cap B$ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ ☐ ☐ ☐ ☐ $P(A^c) = \frac{2}{3}$ | ☐ ☐ ☐ ☐ $P(A^c \cap B)$ ☐ ☐ ☐ ☐ (A and B are events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(A^c) = \frac{2}{3}$. Then the value of $P(A^c \cap B)$ is)
 a. $\frac{7}{4}$
 b. $\frac{3}{8}$
 c. $\frac{5}{12}$
 d. $\frac{5}{8}$

44. $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$ and $|A^2| = 25$ then $|\alpha|$ will be equal to

- a. 25
- b. 5
- c. $\frac{1}{5}$
- d. $\frac{1}{25}$

45. $a \cos 2\theta + a \sin \theta = 2a - 7$ is solvable if the values of a lie in the interval

- a. [2,7]
- b. [2,6]
- c. [3,7]
- d. [5,8]

46. $2^{1+|\cos x|+\cos^2 x+|\cos x|^3} \dots = 4$ (The general solutions of the equation $2^{1+|\cos x|+\cos^2 x+|\cos x|^3} \dots = 4$ are given by)

- a. $n\pi \pm \frac{2\pi}{3}, n \in I$
- b. $n\pi \pm \frac{\pi}{3}, n \in I$
- c. $2n\pi \pm \frac{\pi}{3}, n \in I$
- d. $2n\pi \pm \frac{2\pi}{3}, n \in I$

47. In an entrance test there are multiple choice questions. Each question has four possible options of which one is correct. The probability that a student knows the correct answer is 90%. If he guesses the correct answer, then the probability that he was guessing is)

- a. $\frac{37}{40}$
- b. $\frac{36}{37}$
- c. $\frac{1}{9}$
- d. $\frac{1}{37}$

- a. $(A+Bx)e^{5x}$
- b. $(A+Bx^2)e^{4x}$
- c. $(A+Bx^{-4})e^{-4x}$
- d. $(A+Bx)e^{-4x}$

53. $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$ ☐ x ☐ ☐ ☐ (If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then x will be)

- a. -1
- b. 0
- c. 1
- d. $\frac{1}{2}$

54. a_1, a_2, a_3, a_4 ☐ a_5 ☐ ☐ ☐ ☐ ☐ a_1, a_2, a_3 ☐ ☐ ☐ ☐ ☐ ☐ a_2, a_3, a_4 ☐ ☐ ☐ ☐ ☐ ☐ a_3, a_4, a_5 ☐ ☐ ☐ ☐ ☐ ☐ a_1, a_3, a_5 ☐ ☐ (Let a_1, a_2, a_3, a_4 and a_5 be such that a_1, a_2, a_3 are in A.P. and a_2, a_3, a_4 are in G.P and a_3, a_4, a_5 are in H.P. then a_1, a_3, a_5 are in)

- a. ☐ ☐ ☐ ☐ ☐ ☐ (Arithmetic Progression)
- b. ☐ ☐ ☐ ☐ ☐ ☐ (Harmonic Progression)
- c. ☐ ☐ ☐ ☐ ☐ ☐ (Geometric Progression)
- d. ☐ ☐ ☐ ☐ ☐ ☐ (none of the above)

55. ☐ ☐ ☐ $x+y=5$ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ $x^2 + y^2 + 4x - 6y + 9 = 0$ ☐ ☐ ☐ ☐ ☐ ☐ ☐ (The equation of the circle which touches the line $x+y=5$ at the point $(-2,7)$ and cuts the circle $x^2 + y^2 + 4x - 6y + 9 = 0$ orthogonally, is)

- a. $x^2 + y^2 + 7x + 11y + 38 = 0$
- b. $x^2 + y^2 + 7x - 11y + 38 = 0$
- c. $x^2 + y^2 + 7x - 11y - 38 = 0$
- d. $x^2 + y^2 - 7x - 11y - 38 = 0$

56. $y^2 = 4(x+1)$ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ $x+2y=3$ ☐ ☐ ☐ ☐ ☐ ☐ ☐ (The mirror image of any point on the directrix of the parabola $y^2 = 4(x+1)$ with respect to the line $x+2y=3$ lies on)

- a. $3x+4y-16=0$
- b. $3x+4y+16=0$
- c. $3x-4y+16=0$
- d. $3x-4y-16=0$

57. $\mathbf{r} \cdot (2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}) = 3$ and $\mathbf{r} \cdot (\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) = 7$ (A vector parallel to the line of intersection of the planes $\mathbf{r} \cdot (2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}) = 3$ and $\mathbf{r} \cdot (\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) = 7$ is)

- a. $19\mathbf{i} - 8\mathbf{j} + \mathbf{k} = 3$
- b. $19\mathbf{i} - 8\mathbf{j} - \mathbf{k} = 3$
- c. $19\mathbf{i} + 8\mathbf{j} - \mathbf{k} = 3$
- d. $19\mathbf{i} + 8\mathbf{j} + \mathbf{k} = 3$

58. $x > 0$ (The maximum value of $x^{\frac{1}{x}}$ ($x > 0$) is)

- a. $e^{\frac{1}{e}}$
- b. $\sqrt[e]{e}$
- c. e
- d. $\frac{1}{\sqrt{e}}$

59. $x > 0$ (If $f(x) = 1 + (x-1)e^x$, then, for $x > 0$)

- a. $f(x) < 0$
- b. $f(x) > 0$
- c. $f(x) \leq 0$
- d. $f(x) \geq 0$

60. f and g are inverse functions and $f'(x) = \frac{1}{1+x^3}$ (If g be the inverse of f and $f'(x) = \frac{1}{1+x^3}$, then $g'(x)$ will be)

- a. $1 + 3x^2$
- b. $\frac{1}{1 + [g(x)]^3}$
- c. $\frac{1}{[g(x)]^3 - 1}$
- d. $1 + [g(x)]^3$